

# Discrete R Symmetries and Domain Walls

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## Abstract

Discrete  $R$  symmetries are interesting from a variety of points of view. They raise the specter, however, of domain walls, which may be cosmologically problematic. In this note, we describe some of the issues. In many schemes for supersymmetry breaking, as we explain, satisfying familiar constraints such as suppression of gravitino production, insures that the domain walls are readily inflated away. However, in others, they form after inflation. In these cases, it is necessary that they annihilate. We discuss possible breaking mechanisms for the discrete symmetries, and the constraints they must satisfy so that the walls annihilate effectively.

# 1 Domain Walls and Discrete $R$ Symmetries

If supersymmetry (SUSY) plays a role in low energy physics, discrete symmetries seem likely to be an important component.  $R$  parity is one important example. But more generally, there are a number of reasons to think that *discrete  $R$  symmetries* should play a significant role. For example, approximate, continuous  $R$  symmetries seem an essential component of dynamical supersymmetry breaking, and one way these might arise is as an accidental consequence of discrete  $R$  symmetries. Such symmetries might play a role in suppressing dimension five operators, and accounting for the scale of supersymmetry breaking. Note here that when we speak of  $R$  symmetries, we are excluding simple  $Z_2$  symmetries (like  $R$  parity) which at most rotate the phase of the supercharges by  $\pi$ ; such symmetries can always be redefined by a  $2\pi$  rotation so as to leave the supercharges alone.

Any discrete  $R$  symmetry, however, must be spontaneously broken. This is because a non-vanishing superpotential is required in the effective action at scales of order the supersymmetry breaking scale in order to account for the small value of the cosmological constant, and this breaking must be substantial. Domain walls are then inevitable. These domain walls are problematic cosmologically [1, 2], and either must be inflated away, or, if the symmetry is not exact, must rapidly annihilate [3]. In this note, we consider these issues carefully. We first survey the scales of  $R$  symmetry breaking in different schemes for supersymmetry breaking. We will see instances where other constraints, such as overproduction of gravitinos [4, 5, 6, 7], insure that the discrete symmetry is broken at a scale well above the reheating temperature; then domain walls have been inflated away provided the scale of inflation is not much below the GUT scale. However, in others, the problem is serious; the reheating temperature can be high, restoring the symmetry, or, if not, solving the problem with inflation requires inflation at a rather low scale ( $10^{13}$  GeV or smaller.) So it is necessary to consider the possibility that the walls annihilate, i.e. that the discrete symmetries are not exact. We note that, in string theory, small, explicit breaking of discrete  $R$  symmetries seems common, and we determine conditions under which the domain walls annihilate sufficiently rapidly.

As we will see, in the case of intermediate scale supersymmetry breaking ( $m_{3/2} \sim \text{TeV}$ ), the likely scales of  $R$  breaking range from  $10^{13}$  GeV to  $M_p$ . At the upper end, domain walls are even more catastrophic than conventionally assumed. These walls are parameterically far more problematic than the usual moduli problem. At the lower end, assuming that the usual gravitino problem of such theories is solved, the domain wall problem is readily solved as well. In low scale supersymmetry breaking, the fate of domain walls is a function of the supersymmetry breaking scale. For a broad range of  $m_{3/2}$ , gravitino overabundance is a severe problem [4, 5, 6, 7], and solving that problem requires a relatively low reheating scale, well below the scale of discrete  $R$  breaking. Avoiding domain walls then constrains the value of the Hubble parameter during inflation to be below the  $R$  breaking scale, and one finds again that the domain wall problem is solved without terribly low scale inflation. However, for very low scale gauge mediation, the reheating temperature is not significantly constrained, and the domain wall problem, as we will see, is severe unless the scale of inflation is quite low.

The rest of this paper is organized as follows. In the next section, we discuss the scales of  $R$  symmetry breaking in different scenarios for supersymmetry breaking, and explain under what circumstances domain walls are problematic. In section 3, we consider the problem of domain wall annihilation. We explain that in string theory, it is (in some sense) common for discrete symmetries to be explicitly broken by a small amount, and determine the conditions under which annihilation is sufficiently rapid to avoid cosmological problems. In section 4 we briefly discuss the retrofitted models. In section 5, we consider the possibility that observable gravitational wave signals might emerge from domain wall collisions.

## 2 Scales of R Symmetry Breaking and the Problem of Domain Walls

The universe may have undergone multiple inflationary periods. Assuming that the last inflation has continued for sufficiently long, domain walls are formed in our observable universe if the Hubble parameter during the last inflation  $H_I$  exceeds the  $R$ -breaking scale  $\Lambda$ , or alternatively if the highest temperature of the  $R$ -breaking sector after inflation,  $T_H$ , exceeds  $\Lambda$ <sup>1</sup>. Thus, the walls are formed if the following inequality is met;

$$\max[H_I, T_H] \gtrsim \Lambda. \quad (1)$$

We will see in this section that avoiding domain wall formation, in some cases, is compatible with a relatively large scale of inflation, but the upper bound on the scale can also be quite low, possibly lower than  $10^9$  GeV. Various possibilities for achieving inflation at these scales within supersymmetric models have been discussed in the literature [8, 9, 10, 11].

Without a detailed model, the only information one has about the scale of discrete  $R$  breaking comes from the relation:

$$|\langle W \rangle| = \frac{1}{\sqrt{3}} |F| M_p, \quad (2)$$

where  $F$  is the gravitino decay constant.  $W$  itself is an order parameter of  $R$  symmetry breaking, but potentially there are a variety of order parameters for  $R$  symmetry as well [12]. Roughly speaking, we are interested in two scales. The first we will call  $M_r$ , the scale of  $R$  symmetry breaking. This scale corresponds to the masses of particles which gain mass as a consequence of  $R$  symmetry breaking. The second is  $m_r$ , loosely speaking the mass of the particles whose dynamics is responsible for  $R$  symmetry breaking. If we call

$$\Lambda = |W|^{1/3} \approx 8 \times 10^{12} \text{ GeV} \left( \frac{m_{3/2}}{100 \text{ GeV}} \right)^{1/3}, \quad (3)$$

$\Lambda$  is not necessarily equal to either  $M_r$  or  $m_r$ . Within various standard pictures for supersymmetry breaking, we can enumerate possible values of  $M_r$  and  $m_r$ :

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<sup>1</sup>The highest temperature  $T_H$  is related to the reheating temperature and the inflation scale as  $T_H \simeq (T_R^2 H_I M_p)^{1/4}$ .

1. Intermediate scale supersymmetry breaking (“supergravity breaking”),  $R$  symmetry broken in hidden sector: Here, we can distinguish two cases. Given that we are supposing an underlying discrete  $R$  symmetry, this symmetry might be carried by the hidden sector fields. In this case, we would expect  $M_r = M_p$ ,  $m_r = m_{3/2}$ .
2. Intermediate scale supersymmetry breaking,  $R$  symmetry broken by additional interactions (retrofitting): The discrete  $R$  symmetry might be broken by some other dynamics, as in retrofitted models [13], Then we might have  $M_r = \Lambda = m_r$ .
3. Low scale supersymmetry breaking (gauge mediation): In this case, the  $R$  symmetry is inevitably broken by some additional dynamics at a scale much larger than that of supersymmetry breaking [14, 12, 15]. Within the framework of “retrofitted” models, one might expect that  $M_r = \Lambda = m_r$ . The cosmology of the resulting domain walls is then quite sensitive to  $F$  (or  $m_{3/2}$ ), the supersymmetry breaking order parameter.

In the following subsections, we consider each of these cases in turn.

## 2.1 Intermediate Scale Supersymmetry Breaking

In most scenarios for supersymmetry breaking at an intermediate scale, supersymmetry is broken in a hidden sector. The longitudinal mode of the gravitino is assumed to arise from a chiral field, whose scalar component is a pseudomodulus with mass of order  $m_{3/2}$ . For definiteness, we will describe a situation where there is one such field,  $Z$ . If  $Z$  transforms under the discrete symmetry, then the superpotential has the form, for small  $Z$ ,

$$W = m_{3/2} M_p^2 \left[ \left( \frac{Z}{M_p} \right)^a \left( 1 + c_N \left( \frac{Z}{M_p} \right)^N + \dots \right) \right], \quad (4)$$

where we have assumed that the discrete symmetry is  $Z_N$ . We allow a general Kahler potential consistent with the symmetry. Examining this expression, one sees that in order that the cosmological constant be small, it is necessary that

$$\langle Z \rangle \sim M_p. \quad (5)$$

So the breaking of the discrete symmetry is necessarily of order  $M_p$ . So, indeed,  $M_r \sim M_p$ ,  $m_r \sim m_{3/2}$ .

The domain wall tension in theories of this type is of order  $m_{3/2} M_p^2$ . This can be seen by simple scaling arguments. Cosmologically, this is highly problematic. Models with pseudomoduli with masses of order  $m_{3/2}$  already have severe cosmological problems. When  $H \sim m_{3/2}$ , these moduli simultaneously begin to oscillate and also dominate the energy density of the universe. There are two possible behaviors for the domain walls in such systems:

1. During inflation, the field  $Z$  might be driven to a point in field space, far away from its final stationary point, but at which the discrete symmetry is already broken; the domain walls which

exist at this stage will be inflated away. There is no need for further domain walls to form in the postinflationary dynamics of  $Z$ .

2. During inflation, the field  $Z$  might sit at a point where the discrete symmetry is unbroken. Domain walls, then, *will* form after inflation ends. As the field settles into its minimum, with  $H \approx m_{3/2}$ , one might expect there to be of order one domain wall per horizon. The energy stored in this wall would be of order  $m_{3/2}^{-1} M_p^2$ , corresponding to an energy density of order  $m_{3/2}^2 M_p^2$ , parameterically as large as the energy stored in the field! In other words, at this stage, the domain walls would dominate the energy density. Clearly this is cosmologically unacceptable, unless, somehow, the domain walls can rapidly annihilate. We will discuss this possibility in section 3.

An alternative is that the breaking of the  $R$  symmetry arises in a different sector, as in retrofitted models. The parameter,  $m_{3/2} M_p^2$  in the superpotential might arise through a coupling such as

$$\int d^2\theta \frac{Z W_\alpha^2}{M_p}, \quad (6)$$

where  $W_\alpha$  is the field strength of a new gauge group, with scale  $\Lambda$ . In this case,

$$M_r = m_r = \Lambda, \quad (7)$$

and  $\Lambda = (m_{3/2} M_p^3)^{1/4} \approx 10^{14} \text{ GeV}$  for  $m_{3/2} = 100 \text{ GeV}$ . This scale is quite large <sup>2</sup>; the domain wall tension would be of order  $\Lambda^3$ . But what is most important for the question of domain walls is the value of  $H$  during inflation. Necessarily, in models such as this, the reheat temperature is less than  $10^9 \text{ GeV}$  [6, 7]. So the  $R$ -symmetry breaking phase transition must complete before reheating. The question of whether the domain walls inflate away is the question of whether, during inflation,  $H \equiv H_I$  is greater than  $\Lambda$ . If it is, then the transition may well not be completed during inflation, and the possibility of dangerous domain walls exists. If  $H_I < \Lambda$ , then the domain walls will be inflated away. The latter condition corresponds to an energy scale at inflation,  $E_I \simeq \sqrt{H_I M_p} \approx 10^{16} \text{ GeV}$ , i.e. as long as the scale of inflation is below  $10^{16} \text{ GeV}$ , the domain walls will inflate away.<sup>3</sup>

There is an interesting possibility that the domain walls induce a topological inflation. That is, our universe is contained in a domain wall during the inflation and there will be no walls in the observable universe after inflation. The model is given as follows. We suppose that the discrete  $R$  symmetry is  $Z_{4R}$  and  $X$  and  $Z$  carry the  $R$  charge 2. The superpotential is

$$W = v^2 X \left( 1 - g \left( \frac{Z}{M_P} \right)^2 \right) + \sqrt{g} m_{3/2} M_p Z, \quad (8)$$

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<sup>2</sup>If the  $R$  breaking occurs in the squark condensation instead of gaugino condensation, we can raise the  $R$ -breaking scale  $\Lambda$  up to any very large value. For instance consider  $W = (\bar{Q}Q)^n / M_p^{2n-3}$  with  $\langle \bar{Q}Q \rangle = \Lambda^2$  and the  $R$ -charge of  $(\bar{Q}Q)^n$  equal to 2 modulo  $N$ . Then we get  $\Lambda^{2n} = m_{3/2} M_p^{2n-1}$ . Taking  $n$  sufficiently large we can easily make  $\Lambda > H_I$ .

<sup>3</sup>The WMAP 7-year data sets an upper bound on the inflation scale,  $H_I \lesssim 1.6 \times 10^{14} \text{ GeV}$  [16]. High-scale inflation models such as chaotic inflation [17, 18] satisfying  $H_I > \Lambda$  may produce tensor modes which can be measured by the future CMB observations.

and  $Z$  gets a vev,  $\langle Z \rangle \simeq M_p \sqrt{g}$  [19]. The domain walls are generated in association with the  $R$ -symmetry breaking. We see that the topological inflation occurs in a domain wall if the coupling  $g$  is  $O(1)$  and that the observed density perturbation is explained for  $v = 10^{13} - 10^{15}$  GeV. The difference from the original model of Ref. [19] is that the large superpotential is generated by the Planck scale vev of  $Z$ . The linear term of  $Z$  may be originated from the dynamics which breaks supersymmetry.

Finally, there is the possibility that there are no pseudomoduli fields in the hidden sector. This case could arise if supersymmetry is broken without pseudomoduli, as in the 3-2 model [20]. However, in such a situation, there still must be some additional dynamics responsible for the large  $W$  needed to cancel (the bulk of) the cosmological constant.

To summarize, in the case of intermediate scale supersymmetry breaking, the first question to ask is whether the hidden sector fields are the source of  $R$  symmetry breaking. If they are, the next question is whether the symmetry is already broken during inflation or not. If not, one needs to explore the possibility of domain wall annihilation. In the event that some other dynamics are responsible for breaking the  $R$  symmetry, there need not be a domain wall problem.

## 2.2 Low scale supersymmetry breaking

One of the traditional objections to gauge mediation [21] is that it is hard to understand how one generates the large superpotential necessary to cancel the cosmological constant. If there is a discrete  $R$  symmetry, some additional dynamics, such as gaugino condensation, is needed. As noted in refs. [12, 13], this can be quite natural, if the role of the additional dynamics is to generate the scales in an O’Raifeartaigh model. In such a case, one again has

$$M_r = m_r = \Lambda. \tag{9}$$

(This is the case even if the model is not retrofitted, and one simply has gaugino condensation of something similar to generate  $\langle W \rangle$  [14].) So we must ask how large is  $\Lambda$  for a given scale of supersymmetry breaking, and then determine for what value of  $E_I$  the resulting domain walls are inflated away.

Let’s start with extremes. Consider a relatively high scale for supersymmetry breaking,  $m_{3/2} = 100$  MeV. In this case,  $\Lambda = (m_{3/2} M_p^2)^{1/3} \approx 10^{12}$  GeV. In such a case, the gravitino overabundance already requires that the  $T_R < 10^6$  GeV. So, again, the transition to the broken symmetry phase must occur before reheating. Now requiring  $m_r > H_I$  leads to  $E_I < 10^{16}$  GeV. So rather high scales of inflation are permitted. At the other extreme, suppose  $m_{3/2} \approx 10$  eV. In this case,  $\Lambda \approx 10^9$  GeV. In this case, there is no significant constraint on the reheating temperature, so one can be concerned that  $T_R > \Lambda$ . Even if not, the condition on the scale of inflation is now  $E_I < 10^{14}$  GeV.

### 3 Explicit R Breaking and the Fate of Domain Walls

In cases where inflation occurs before the formation of domain walls, their disappearance might be explained by explicit breaking of the symmetry [3, 22]. It might seem troubling to postulate a symmetry, and then invoke small, explicit breaking, but this phenomenon is rather common in string theories, where discrete symmetries (and discrete  $R$  symmetries in particular) are often anomalous [23].<sup>4</sup> In these cases, there is typically an axion-like field which transforms non-linearly under the discrete symmetry, whose couplings cancel the would-be anomaly. Thus there is an exact symmetry, which is spontaneously broken at a high scale, and an approximate symmetry at low energies. The effects of the breaking at low energy are exponentially small if appropriate couplings are small. As the scale of spontaneous breaking of the exact symmetry could readily be the Planck scale or the GUT scale, one does not need to worry about domain walls arising from the breaking of the underlying, exact symmetry. The breaking of the approximate discrete symmetry at low energies has the potential to produce problematic domain walls. On the other hand, the exponentially small effects associated with the anomaly will generate a small splitting in the energies of the different domains. The question, then, is how large is the splitting and how quickly the domain walls annihilate.

An essentially equivalent phenomenon can occur if, for example, there are two gaugino condensates, one breaking a symmetry  $Z_N$  at the scale  $\Lambda$ , and another a symmetry  $Z_{N'}$  at a lower scale  $\Lambda'(< \Lambda)$ . From a more microscopic point of view, these symmetries are incompatible (all gauginos must transform under any  $R$  symmetry), and should again be thought of as anomalous. One can view the gaugino condensate of the higher scale theory as accounting for the size of  $\langle W \rangle$ , while the other lower scale condensate generates the mass splitting.

In either case, we can represent the effects of the explicit  $R$ -symmetry breaking through a constant,  $w_c$ , in the superpotential, and consider the effects of the spontaneous breaking to go as  $W_0 = \Lambda^3 \alpha^k$ , where  $\Lambda$  is some dynamical scale,  $\alpha$  is a suitable root of unity, and  $\Lambda^3 > w_c$ . Domain walls are produced at the phase transition associated with the scale  $\Lambda$ , and they will annihilate as a result of the explicit breaking  $w_c$ . Calling  $w_c = a m_{3/2} M_p^2$ , and  $\Lambda^3 = b m_{3/2} M_p^2$ , we have  $a + b = 1$ . (Here and in what follows we drop  $\alpha^k$ .) We take  $a \ll b \simeq 1$ .

The splitting between states then behaves as

$$\epsilon = a b m_{3/2}^2 M_p^2. \quad (10)$$

Now the value of  $H$  when the walls collide can be estimated as follows. Calling  $x$  a wall coordinate, and adopting the notation of Vilenkin [3] in which  $\sigma$  is the wall tension, we have

$$\sigma \ddot{x} = \epsilon \quad (11)$$

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<sup>4</sup>If it is not anomalous, one may consider a gauged  $Z_{NR}$  symmetry. In this case the domain walls are not formed, or, even if they are formed, they will disappear in the end. However it is not easy to have such a non-anomalous  $Z_{NR}$  symmetry [24].

and  $\sigma \approx b m_{3/2} M_p^2$ .

$$x \approx \frac{1}{2} \frac{\epsilon}{\sigma} t^2 \quad (12)$$

giving, for the condition  $x \approx H^{-1}$ ,

$$H \approx a m_{3/2}. \quad (13)$$

The requirement that, at this time, the Schwarzschild radius  $r_s$  associated with the domain wall tension in a horizon be smaller than the horizon gives the condition:

$$H^{-1} \gg r_s \quad (14)$$

or

$$H \gg b m_{3/2} \quad (15)$$

from which we have

$$b \ll a \simeq 1. \quad (16)$$

This contradicts our original assumption. A picture in which domain walls form with characteristic scale  $\Lambda$ , and annihilate due to some smaller, explicit  $R$  breaking, is not viable. At best, walls connected with the lower scale of symmetry breaking can annihilate as a result of the splitting between the domains generated by the higher scale dynamics.

As an example, consider the case of two gaugino condensates, one associated with scale  $\Lambda$ , one with  $\Lambda'$ . if  $w_c$  arises from gaugino condensation at a scale  $\Lambda'$  (i.e.,  $w_c \sim \Lambda'^3$ ), there are two kinds of walls. The precise value of the mass splitting then depends on the combination of the two vacua. In the above example with  $Z_N$  and  $Z_{N'}$ , the typical magnitude of the bias is still given by (10), and there is a unique vacuum if  $N$  and  $N'$  are coprime with respect to each other. The lower scale domain walls annihilate when  $H \sim b m_{3/2}$ , which is earlier than (13)<sup>5</sup>. This is because the walls with a smaller tension annihilate earlier for the same splitting. The subsequent dynamics of the higher scale walls are the same as described above.

## 4 Domain Walls in Gauge Mediation

Gauge mediated models with retrofitting as the origin of supersymmetry breaking raise similar issues to those in the gravity mediated case (with retrofitting). Again, one seems to require that the explicit breaking be rather large.

In [12], it was argued that a natural way in which to understand dynamical supersymmetry breaking was to suppose that gaugino condensation (in a generalized sense discussed there) at a scale of order

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<sup>5</sup>Even if some of the lower scale walls do not disappear at this time, they do not affect the subsequent evolution of the higher scale walls.



$(FM_p)^{1/3}$  generated a dynamical scale responsible for supersymmetry breaking (with Goldstino decay constant  $F$ ). This naturally correlated the scale of supersymmetry breaking and the need for a large  $\langle W \rangle$  needed to obtain a small cosmological constant [14]. But we see that if the domain walls associated with this condensation are to be eliminated through an explicit breaking of the symmetry, the scale of these “retrofitting” interactions must be lower than  $(M_p F)^{1/3}$ .

It is worth recalling the basic ideas of retrofitted models. Here one starts with, say, a conventional O’Raifeartaigh model,

$$X(A^2 - \mu^2) + mAY \quad (17)$$

and accounting for the scales  $\mu^2$  and/or  $m$  by a coupling of the fields to some set of interactions which dynamically generate a scale, typically breaking a discrete  $R$  symmetry, e.g.

$$\frac{XW_\alpha^2}{M} + XA^2 + \frac{S^2}{M}AY \quad (18)$$

The interactions associated with  $W_\alpha^2, S$ , generate the terms  $\mu^2$  and  $m$ , and also an expectation value for  $W$ , of order  $FM$ . It would seem natural to identify  $M$  with  $M_p$ , but in order to destroy domain walls, it is necessary that  $M$  (and  $\langle W_\alpha^2 \rangle$ ) be smaller than that. It is the other interactions, responsible for the disappearance of the domain walls, which must generate the term which gives small cosmological constant.

## 5 Gravity Waves From Domain Wall Collisions

Domain walls generally produce gravity waves when they collide and disappear. In this section we estimate the abundance and frequency of the gravity waves, following Ref. [25].

In the violent collisions of domain walls, gravity waves are produced at a frequency  $f_p$  corresponding to a typical physical length scale. One of the important scales is the curvature radius of the walls. The domain-wall network is known to follow scaling solution [26, 27], and then a typical curvature radius is the Hubble horizon. Also, since the domain-wall energy density decreases more slowly than radiation, most of the gravity waves are produced when the walls disappear. Therefore we expect that the gravity waves are produced at  $f_p \sim H_d$ , where  $H_d$  is the Hubble parameter at the disappearance of the walls. Whether or not the gravity waves with frequencies greater than  $H_d$  are produced depends on the domain-wall dynamics at the sub-horizon scales. In particular, there is another length scale, the wall width,  $\Delta$ , which might affect the spectrum. According to the numerical simulation [28], the gravity waves from the walls have a broad and comparatively flat spectrum, ranging from  $f_p \sim H_d$  to  $f_p \sim \Delta^{-1}$ , with an intensity consistent with that obtained from a simple dimensional analysis.

The frequency is red-shifted due to subsequent cosmic expansion, and so, the frequency we observe

today,  $f_0$ , is much smaller than that at the production,  $f_p$ :

$$f_0 \simeq 1 \times 10^2 \left( \frac{g_*}{200} \right)^{-\frac{1}{12}} \left( \frac{H_d}{1 \text{ GeV}} \right)^{-\frac{1}{2}} \left( \frac{f_p}{1 \text{ GeV}} \right) \text{ Hz}, \quad (19)$$

where  $g_*$  counts the relativistic degrees of freedom. Here, radiation domination is assumed.

The intensity of the gravitational waves decreases as the universe expands, since the amplitude too is red-shifted for sub-horizon modes. In order to characterize the intensity, it is customary to use a dimensionless quantity,  $\Omega_{\text{gw}}(f)$ , defined by

$$\Omega_{\text{gw}}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{gw}}}{d \log f}, \quad (20)$$

where  $\rho_{\text{gw}}$  is the energy density of the gravitational waves,  $\rho_c$  the critical energy density, and  $f$  the frequency. Let us estimate the magnitude of  $\Omega_{\text{gw}}(f)$ . The energy of the gravitational waves in a horizon at the disappearance of the walls is estimated by

$$E_{\text{gw}} \sim G \frac{M_{DW}^2}{R_*} \sim \frac{\sigma^2}{H_d^3 M_p^2}, \quad (21)$$

where  $G = 1/(8\pi M_p^2)$  is the Newton constant,  $M_{DW}$  the energy stored in the domain walls, and  $R_*$  the typical spatial scale of the energy distribution. In the second equality in (21), we have used  $M_{DW} \sim \sigma/H_d^2$  and  $R_* \sim 1/H_d$ , where  $H_d \sim \epsilon/\sigma$  is the Hubble parameter when the walls disappear. If the reheating is completed before  $H = m_{3/2}$ , we have

$$\Omega_{\text{gw}}(f_0) h^2 \sim 10^{-5} \left( \frac{\sigma^2}{\epsilon M_p^2} \right)^2, \quad (22)$$

where  $h$  is the present Hubble parameter in units of 100 km/s/Mpc. Note that the condition for the domain walls to disappear before the domination, (14), is equivalent to  $\sigma^2/\epsilon M_p^2 \ll 1$ .

So far, radiation domination has been assumed. The gravitino problem implies however that the reheating temperature cannot be arbitrarily high [6, 7, 5]. If the gravity waves are produced before the reheating,  $f_0$  is shifted to a smaller value and the intensity is weakened. Specifically, assuming that the inflaton behaves like non-relativistic matter before the reheating, both the frequency and intensity are suppressed by a factor of  $(T_R^2/H_d M_p)^{3/2}$ , where  $T_R$  is the reheating temperature.

Here let us briefly mention the sensitivities of the ongoing and planned experiments on gravitational waves. One of the ground-based experiments, LIGO [29], is in operation and it is sensitive to the frequency between  $O(10)$  Hz and  $10^4$  Hz. The latest upper bound is  $\Omega_{\text{gw}} h^2 < 6.5 \times 10^{-5}$  around 100 Hz [30], and an upgrade of the experiment, Advanced LIGO [31, 32], would reach sensitivities of  $O(10^{-9})$ . The sensitivity of LCGT [33] would be more or less similar to that of Advanced LIGO. There are also planned space-borne interferometers such as LISA [34], BBO [35] and DECIGO [36]. LISA is sensitive to the band of  $(0.03 - 0.1) \text{ mHz} \lesssim f_0 \lesssim 0.1 \text{ Hz}$ , and it can reach  $\Omega_{\text{gw}} h^2 < 10^{-12}$  at  $f_0 = 1 \text{ mHz}$ . Moreover, BBO and DECIGO will cover  $10 \text{ mHz} \lesssim f_0 \lesssim 10^2 \text{ Hz}$  with much better sensitivity.

Let us take the example considered in Sec. 3, i.e., the domain walls associated with the gaugino condensation at  $\Lambda$  is destroyed due to a bias  $\epsilon$  induced by a constant  $w_c$  [25]. In this case the walls disappear at  $H_d \sim m_{3/2}$ . Here we assume  $b \ll a \sim 1$ . The gravity waves have a broad spectrum from  $f \sim 1 \text{ kHz} (m_{3/2}/100 \text{ GeV})^{1/2}$  to  $f \sim 10^{14} \text{ Hz } b^{1/3} (m_{3/2}/100 \text{ GeV})^{-1/6}$ , with an intensity  $\Omega_{\text{gw}} h^2 \sim 10^{-5} b^2$ . Therefore, for a light gravitino mass  $m_{3/2} \lesssim 100 \text{ GeV}$  and a moderately large value of  $b (\ll 1)$ , the frequency and intensity may fall in the range of the future gravity wave experiments. In the second example, it is the domain walls associated with the gaugino condensate, which gives a small cosmological constant, disappear due to an explicit  $R$ -breaking induced by another gaugino condensate,

$$\int d^2\theta \frac{S}{M} (W_\alpha^2 + W_\alpha'^2), \quad (23)$$

where  $S$  is a singlet, and  $M \ll M_p$ . In this case the tension and bias are given by  $\sigma = \Lambda^3 \simeq m_{3/2} M_p^2$  and  $\epsilon \simeq \Lambda^3 \Lambda'^3 / M^2 \equiv c m_{3/2}^2 M_p^2$ . In order for the walls to disappear before they dominate the energy density of the Universe,  $c$  must be greater than 1. The walls decay when  $H_d \sim \Lambda'^3 / M^2 = c m_{3/2}$ . The gravity wave spectrum ranges from  $f \sim 1 \times 10^2 \text{ Hz } c^{1/2} (m_{3/2}/\text{GeV})^{1/2}$  to  $f \sim 2 \times 10^{14} \text{ Hz } (m_{3/2}/\text{GeV})^{-1/6}$ , with an intensity  $\Omega_{\text{gw}} h^2 \sim 10^{-5} c^{-2}$ .

To summarize this section, gravity waves are generally produced when domain walls collide and disappear. The frequencies of these gravity waves happens to be close to those covered by the ongoing and planned gravity-wave experiments. Considering the sensitivities of future experiments, the gravity waves from domain-wall decay may be detectable, for certain parameters (e.g., not too small  $b$  or  $c^{-1}$  in the cases considered above).

## 6 Conclusions

There are a number of reasons to believe that discrete  $R$  symmetries may play an important role in low energy supersymmetry. Perhaps most dramatically, the smallness of  $\langle W \rangle$  can be naturally explained by a spontaneously broken discrete  $R$  symmetry, but such symmetries are alsolikely to play a role in supersymmetry breaking, and may well be important in understanding the suppression of rare process. Such discrete symmetry breaking implies the existence of domain walls. Because of the role of  $W$  as an order parameter for  $R$  symmetry breaking, and in accounting for the small value of the cosmological constant, one has some idea of the tension of these walls. We have seen that adopting a picture for supersymmetry breaking then determines whether these walls may or may not be produced before the end of inflation. In many scenarios, the domain walls can be inflated away even by inflation at scales comparable to the GUT scale, but in many others, they are produced after inflation. In this case, they must somehow disappear. This can be accomplished if there is a large enough explicit  $R$  breaking, and such breaking is plausible if one examines typical string vacua with discrete  $R$  symmetries. In such a case, there is typically a non-anomalous symmetry broken at a very high scale, and an approximate symmetry broken at a lower scale. It is the domain walls associated with the lower scale breaking that are the greatest danger. But requiring that the walls annihilate before gravitational collapse provides

strong constraints. The interactions responsible for the explicit  $R$  breaking must, in particular, make the dominant contribution to the superpotential. In the case that high scale instantons or analogous effects are responsible for the explicit breaking, these must also give the dominant contribution to  $W$ ; similarly, in the case of two gaugino condensates, the couplings of the higher scale condensate must be suppressed by a scale smaller than the Planck scale. If such domain walls were produced at an early stage, we have seen that their annihilations have implication for future gravity-wave experiments.

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